

NON-SINGULAR FOUR-DIMENSIONAL BLACK HOLES AND THE JACKIW-TEITELBOIM THEORY

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ABSTRACT

A four-dimensional dilaton-gravity action whose spherical reduction to two dimensions leads to the Jackiw-Teitelboim theory is presented. A nonsingular black hole solution of the theory is obtained and its physical interpretation is discussed. The classical and semiclassical properties of the solution and of its two-dimensional counterpart are analysed. The two-dimensional theory is also used to model the evaporation process of the near-extremal four-dimensional black hole. We describe in detail the peculiarities of the black hole solutions, in particular the purely topological nature of the Hawking radiation, in the context of the Jackiw-Teitelboim theory.

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1. Introduction.

In the last years dilaton-gravity theories have been widely investigated both in two and four spacetime dimensions [1-6]. In particular the model proposed by Callan, Giddings, Harvey and Strominger (CGHS) [1] has generated new interest and activity on black hole physics. It was recognised that two-dimensional dilaton-gravity models represent a theoretical laboratory for investigating the fundamental issue of information loss in the black hole evaporation process. Moreover the CGHS model can be viewed as the low-energy effective theory that describes the S-wave sector in the evaporation process of the near-extreme magnetically charged dilaton black hole in four dimensions. The four-dimensional black hole is also of interest because it is a classical solution of low-energy effective string field theory [3,4].

In a previous paper [5] we found four-dimensional dilaton black hole solutions which can be considered as a generalization of the Garfinkle-Horowitz-Strominger (GHS) solutions [3,4]. They are also classical solutions of a field theory that arises as a low-energy approximation to string theory. The corresponding two-dimensional effective theory that describes a magnetically charged four-dimensional black hole near its extremality represent a generalization of the CGHS model [6]. In this paper we will study in detail another special case of these models, namely the one whose dimensional reduction to two dimensions leads to the Jackiw-Teitelboim (JT) theory [7]. The relevance of this model is twofold. From a purely four-dimensional point of view the corresponding magnetically charged black hole solutions evidenciate properties which are in some sense intermediate between the Reissner-Nordström (RN) and the GHS black holes and in addition enjoy the peculiar property of being free from curvature singularities. On the other hand the spherical reduction to two dimensions leads to the JT theory. The latter can therefore be used to model the S-wave scattering of the 4d black hole near extremality. In this context the dynamics of the JT theory is very peculiar. In fact the black hole solutions of the JT theory present features which have no correspondence in other theories. Their existence in the context of the theory is related to the choice of the boundary conditions for the spacetime, they have in some sense a purely topological origin. As a consequence the Hawking evaporation process is a purely topological effect. Our results are also interesting with respect to the

black hole solutions in three dimensions. It has been demonstrated [8] that the JT theory arises from dimensional reduction of the black hole solutions of Bañados, Teitelboim and Zanelli (BTZ) [9]. Thus our results about the black hole physics in the JT theory can be as well used to model the evaporation process of a three-dimensional black hole.

The structure of the paper is as follows. In sect. 2 we study the four-dimensional model. In particular we derive the magnetically charged black hole solutions, we study their local and global properties and their behaviour in the extremal limit. In sect. 3 we consider the two-dimensional theory obtained by spherical reduction of the four-dimensional one. The classical properties of the corresponding black hole solutions are analysed at length in particular in connection with the four-dimensional ones. In sect. 4 we study the semiclassical behaviour of the two-dimensional theory, in particular the evaporation process of the black hole including the back reaction. The main results of our investigation are summarized in sect 5.

2. The four-dimensional model.

Let us consider the 4d action:

$$S = \int \sqrt{g} d^4x e^{-2\phi} (R - F^2), \quad (2.1)$$

where $F_{\mu\nu}$ describes a Maxwell field and ϕ a scalar (dilaton).

The relevance of this action resides in the fact that its dimensional reduction to two dimensions leads, as we shall see in the following, to the Jackiw-Teitelboim theory. Moreover, it is a special case ($k = 0$) of the low energy effective string actions discussed in [5,6]. A conformal transformation of the metric field $\hat{g}_{\mu\nu} = e^{-2\phi} g_{\mu\nu}$ leads to the minimally coupled action:

$$S = \int \sqrt{\hat{g}} d^4x [\hat{R} - 6(\hat{\nabla}\phi)^2 - e^{-2\phi} F^2]. \quad (2.2)$$

However, we prefer to discuss the action in the form (2.1). One of the reasons is that the solutions of (2.1) describe non-singular black holes similar to those discussed in [8,10] for the JT theory.

The field equations stemming from (2.1) are:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2F_{\mu\rho}F_{\nu}^{\rho} - \frac{1}{2}g_{\mu\nu}F^2 + e^{2\phi}[(\nabla_{\mu}\nabla_{\nu} - g_{\mu\nu}\nabla^2)e^{-2\phi}],$$

$$R = F^2, \quad (2.3)$$

$$\nabla_\mu (e^{-2\phi} F^{\mu\nu}) = 0.$$

A spherically symmetric black hole solution of the field equation is given [6] by a magnetic monopole:

$$F_{ij} = \frac{Q_M}{r^2} \epsilon_{ij},$$

with metric

$$ds^2 = - \left(1 - \frac{r_+}{r}\right) dt^2 + \left(1 - \frac{r_+}{r}\right)^{-1} \left(1 - \frac{r_-}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (2.4)$$

and dilaton field

$$e^{2(\phi - \phi_0)} = \left(1 - \frac{r_-}{r}\right)^{-1/2}. \quad (2.5)$$

The two parameters r_+ and r_- ($r_+ \geq r_-$) are related to the charge Q_M and to the mass M of the black hole by the relations:

$$2M = r_+, \quad Q_M^2 = \frac{3}{4} r_+ r_-. \quad (2.6)$$

The temperature and entropy of the solution are given respectively by [6]:

$$T = \frac{1}{4\pi r_+} \left(1 - \frac{r_-}{r_+}\right)^{1/2} = \frac{(M^2 - Q_M^2/3)^{1/2}}{8\pi M^2}, \quad S = \pi r_+^2 = 4\pi M^2. \quad (2.7)$$

The temperature vanishes in the extremal limit $r_+ = r_-$, i.e. $M^2 = \frac{1}{3} Q_M^2$, which should therefore be considered as the ground state for the Hawking evaporation process of a black hole of given charge.

The spatial sections of the metric coincide with those of the Reissner-Nordström solution of general relativity and those of the Garfinkle-Horowitz-Strominger solution of the low-energy effective action of string theory [3,4]. Owing to the difference in the g_{00} component, however, the three metrics possess quite different physical properties.

The metric (2.4) is asymptotically flat and the curvature is regular everywhere except at $r = 0$. This point, however, is not part of the manifold, since the range of the radial coordinate is given by $r \geq r_-$. In fact, even if the manifold is regular at $r = r_-$ (a coordinate singularity is placed at this point, but the curvature tensor is regular), for

$r < r_-$ the metric becomes euclidean and the dilaton imaginary, so that one is forced to cut the manifold at $r = r_-$. Furthermore, a horizon is present at $r = r_+$. Hence, the solution (2.4) describes a non-singular four-dimensional black hole. This is reminiscent of the regular two-dimensional black hole discussed in [8,10]. For negative values of r_+ and r_- , instead, one has negative mass and a naked singularity.

A better understanding of the causal structure of the spacetime can be obtained by discussing its maximal extension and the Penrose diagram. Let us thus consider only the two-dimensional $r - t$ sections of the metric and introduce the "Regge-Wheeler tortoise" coordinate, defined by:

$$dr_* = \left(1 - \frac{r_+}{r}\right)^{-1} \left(1 - \frac{r_-}{r}\right)^{-\frac{1}{2}} dr. \quad (2.8)$$

The change of variables $u = t - r_*$, $v = t + r_*$ takes the metric in the form

$$ds^2 = - \left(1 - \frac{r_+}{r}\right) dudv, \quad (2.9)$$

with r defined implicitly in terms of u and v by (2.8). In these coordinates the metric is clearly regular at $r = r_-$. One can now perform another change of coordinates which eliminates the singularity at $r = r_+$:

$$U = -\exp(-\beta u), \quad V = \exp(\beta v),$$

where $\beta = \frac{1}{2} \sqrt{\frac{r_+ - r_-}{r_+^3}}$. The final result is the metric expressed in the Kruskal form:

$$ds^2 = - \frac{(r_+)^{-(1+\gamma/2)}}{\beta^2 r} e^{-2\beta\sqrt{r(r-r_-)}} \left(\sqrt{r-r_-} + \sqrt{r}\right)^\gamma \times \left(\sqrt{r_+(r-r_-)} + \sqrt{r(r_+-r_-)}\right)^2 dU dV \quad (2.10)$$

with

$$\gamma = -2\beta(2r_+ + r_-), \quad UV = -\exp(2\beta r_*), \quad \frac{V}{U} = -\exp(2\beta t).$$

This form of the metric is regular both at r_+ and r_- . In a standard way one can deduce from it the form of the Penrose diagram (fig.1), which results analogous to that of

the Schwarzschild solution. The only difference is that the singularity is now replaced by an inner horizon, which represents the boundary of the manifold.

In the extremal case one can proceed in a similar way. However, we have not been able to find an explicit form for the Kruskal metric. The Penrose diagram is given in fig. 2 and is identical to that of the GHS solution.

In the following, we shall be especially interested in the properties of the extremal limit $r_+ = r_-$. For this purpose is useful to define a new coordinate η , such that $\eta = \text{arcsinh} \sqrt{\frac{r-r_+}{r_+-r_-}}$. In terms of η , the metric and the dilaton field (2.4), (2.5) take the form

$$ds^2 = -4Q^2 \frac{\Delta \sinh^2 \eta}{r_+ + \Delta \sinh^2 \eta} dt^2 + (r_+ + \Delta \sinh^2 \eta)^2 (4d\eta^2 + d\Omega^2),$$

$$e^{2(\phi-\phi_0)} = \left[\frac{r_+ + \Delta \sinh^2 \eta}{\Delta \cosh^2 \eta} \right]^{\frac{1}{2}}, \quad (2.11)$$

where $\Delta = r_+ - r_-$ and $Q = (2/\sqrt{3})Q_M$.

In the limit $\Delta \rightarrow 0$, the spatial sections of our solution can be described as an asymptotically flat region attached to an infinitely long tube (the "throat"), in analogy with the RN and GHS case. It is not possible, however, to describe the extremal limit by a unique metric: rather, there are several regimes under which the limit can be approached, which correspond to different solutions of the action (2.1).

The region of the throat near the horizon is described, for positive r_+ and r_- , by the solution:

$$ds^2 = -4Q^2 \sinh^2 \eta dt^2 + Q^2 (4d\eta^2 + d\Omega^2),$$

$$e^{2(\phi-\phi_0)} = \frac{Q}{\cosh \eta}. \quad (2.12)$$

In this limit the metric is the direct product of a 2d spacetime of constant negative curvature and a two-sphere of radius Q .

The extremal limit of the negative mass solution, with negative r_+ and r_- and naked singularity is given instead by:

$$ds^2 = -4Q^2 \cosh^2 \eta dt^2 + Q^2 (4d\eta^2 + d\Omega^2),$$

$$e^{2(\phi-\phi_0)} = \frac{Q}{\sinh \eta}, \quad (2.13)$$

It appears that in the extremal limit also the negative mass solution becomes regular: the metric (2.13), in fact, is the direct product of a 2d spacetime of constant negative curvature

and a two-sphere. We shall discuss in more detail the properties of this solution in the following.

Both solutions (2.12) and (2.13) tend, for $\eta \rightarrow \infty$ to the metric

$$ds^2 = -4Q^2 e^{2\eta} dt^2 + Q^2 (4d\eta^2 + d\Omega^2), \quad (2.14)$$

with linear dilaton:

$$\phi = -\frac{1}{2}\eta,$$

which is a direct product of 2d anti-de Sitter spacetime and a two-sphere and corresponds to the throat region.

Finally, the asymptotically flat region and the throat can be described by the solution:

$$\begin{aligned} ds^2 &= -4Q^2 \left(1 + \frac{Q}{y}\right)^{-1} dt^2 + \left(1 + \frac{Q}{y}\right)^2 (dy^2 + y^2 d\Omega^2), \\ e^{2(\phi - \phi_0)} &= \left(1 + \frac{Q}{y}\right)^{\frac{1}{2}} \end{aligned} \quad (2.15)$$

and $y = r + Q \geq 0$. This metric is everywhere regular and describes the transition between an asymptotically flat spacetime for $y \rightarrow \infty$ and one with topology $H^2 \times S^2$ for $y \rightarrow 0$, H^2 being 2d anti-de Sitter spacetime .

An unpleasant property of the solution (2.4) is that, contrary for example to the GHS solution, it is not geodesically complete even in the extremal limit. This is easily seen by considering the geodesic equation for the radial motion:

$$\left(\frac{dr}{d\lambda}\right)^2 = \left(1 - \frac{r_-}{r}\right) \left[E^2 - \left(1 - \frac{r_+}{r}\right)\epsilon\right], \quad (2.16)$$

where $\epsilon = 0, 1, -1$ for light-like, time-like, space-like geodesics respectively, E is the energy of the orbit and λ is the affine parameter. The boundary at $r = r_-$ is in general at finite distance, but becomes in the extremal case infinitely far away along time-like and space-like geodesics. In spite of this, light-like geodesics have finite length also in the extremal limit. In fact, for $\epsilon = 0$, the geodesic length is given by

$$\lambda = \frac{1}{E} \left[\sqrt{r(r - r_-)} + r_- \ln (\sqrt{r - r_-} + \sqrt{r}) \right], \quad (2.17)$$

which is always finite for $r \geq r_-$. The length of time-like geodesics, instead, is finite for $r_+ > r_-$, but tends to infinity in the extremal limit.

This should be compared with the RN and GHS solutions, which, as already remarked, have the same spatial sections as our solution. In the extremal RN metric, the horizon is at finite distance both along time-like and light-like paths, while in the extremal GHS metric the distance is infinite in both cases. Space-like geodesics have instead infinite length for RN and GHS extremal black holes. Our solution is therefore in some sense intermediate between the two. The lack of geodesic completeness of our solution can be however remedied if one glues to the boundary of the spacetime at $r = r_-$ another copy of the same manifold. This is possible because the extrinsic curvature vanishes there.

3. The two-dimensional model.

The two-dimensional action of the JT theory can be obtained by spherical reduction of the four-dimensional action (2.1) for a near-extreme magnetically charged black hole solution. This dimensional reduction has been described in [6] in the context of a general four-dimensional dilaton gravity theory of which the action (2.1) represents just a special case. From (2.1), taking the angular coordinates to span a 2-sphere of constant radius Q , we get the dimensionally reduced action:

$$S = \frac{1}{2\pi} \int \sqrt{g} d^2x e^{-2\phi} [R + 2\Lambda^2]. \quad (3.1)$$

where Λ is related to the four-dimensional magnetic charge by $\Lambda = (2Q)^{-1}$. The action (3.1) describes the JT theory [7].

The general time-independent solution of the corresponding field equations is by now well-known and has been discussed at length in the literature [11,6,8,10]. In Schwarzschild coordinates it has the form:

$$ds^2 = -(\Lambda^2 r^2 - a^2) dt^2 + (\Lambda^2 r^2 - a^2)^{-1} dr^2, \quad e^{2(\phi - \phi_0)} = (\Lambda r)^{-1}. \quad (3.2)$$

where a^2 is an integration constant which can assume both positive and negative values and is related to the Arnowitt-Deser-Misner (ADM) mass of the solution by

$$M = \frac{1}{2} e^{-2\phi_0} a^2 \Lambda. \quad (3.3)$$

The ADM mass is calculated with respect to the asymptotic solution with $a = 0$. Independently of the value of the parameter a^2 the solution (3.2) describes a spacetime of constant negative curvature $R = -2\Lambda^2$, i.e two-dimensional anti-de Sitter space. The solutions (3.2) with a^2 positive, negative or zero describe in Schwarzschild coordinates the two-dimensional section of the extremal four-dimensional solutions (2.12), (2.13), (2.14) respectively [6]. For $a^2 > 0$ the spacetime (3.2) has an horizon at $r = a/\Lambda$ indicating that it can be interpreted as a two-dimensional black hole, whereas for $a^2 < 0$ the solution has negative ADM mass¹. However the metrics with different values of the parameter a^2 represent different parametrizations of the same manifold, namely anti-de Sitter spacetime, with coordinate patches covering different regions of the space. This fact was first demonstrated, for the metrics with $a^2 > 0$, in [8] (see also [6]). Indeed one can easily find the coordinate transformations relating the different metrics. The transformation

$$\begin{aligned} r &\rightarrow a\Lambda t r, \\ e^{2a\Lambda t} &\rightarrow \Lambda^2 t^2 - \Lambda^{-2} r^{-2}, \end{aligned} \tag{3.4}$$

brings the metric with $a^2 > 0$ into the metric with $a = 0$, whereas

$$\begin{aligned} r &\rightarrow \frac{a}{2} (\Lambda^{-2} r^{-1} - \Lambda^2 r t^2 - r), \\ \tan(a\Lambda t) &\rightarrow \frac{1}{2} (\Lambda t - \Lambda^{-3} r^{-2} t^{-1} - \Lambda^{-1} t^{-1}), \end{aligned} \tag{3.5}$$

relates the metric with $a^2 < 0$ to the metric with $a = 0$. Notice that even though the solutions (3.2) locally describe the same spacetime independently of the value of a^2 , the expression for the dilaton becomes different for the three cases after using (3.4) and (3.5). To distinguish different solutions in (3.2) we will denote with ADS^+ , ADS^0 and ADS^- the spacetimes corresponding to positive, zero and negative a^2 respectively.

The equivalence of the metric part of the solutions (3.2) with different values of a^2 up to a coordinate transformation makes it difficult to interpret ADS^+ as a black hole. In fact the spacetime can be extended beyond $r = 0$, and the maximally extended spacetime

¹ The negative mass is related to the choice of the ground state which we have identified with the solution with $a = 0$. The reason for this choice will be explained later on this section.

is the whole of two-dimensional anti-de Sitter spacetime which of course has no horizons and is geodesically complete [8]. However ADS^+ can represent a black hole if we cut the spacetime off at $r = 0$. The reason why one has to cut off the spacetime at this point is clear if one takes into account the expression (3.2) for the dilaton. By analytically continuing the spacetime beyond $r = 0$ one would enter in a region where $\exp(-2\phi)$ becomes negative. The two-dimensional action (3.1) has been obtained by spherical reduction of the four-dimensional action (2.1). When this reduction is carried out, the area of the transverse sphere of constant radius in four dimensions becomes the factor $\exp(-2\phi)$ multiplying the Ricci scalar in (3.1) and has therefore to be positive. Thus if one wants to model a four-dimensional near-extremal magnetic black hole by means of a two-dimensional solution of the action (3.1) one has to cut the spacetime off at $r = 0$. The Penrose diagram for ADS^+ and ADS^0 are shown in fig. 3 and 4. Notice the very peculiar role played by the dilaton in the context of the JT theory: it sets the boundary conditions on the spacetime, making solutions which have the same local properties topologically not equivalent.

Once one has recognised ADS^+ as a black hole one can use it to model the S-wave sector of the evaporation process of a four-dimensional black hole near its extremality. In particular one can associate to it thermodynamical parameters. Using standard formulae we have for the temperature and entropy of the hole:

$$\begin{aligned} T &= \frac{a\Lambda}{2\pi} = \frac{1}{2\pi} e^{\phi_0} (2M\Lambda)^{1/2}, \\ S &= 4\pi e^{-\phi_0} (M/2\Lambda)^{1/2} = 2\pi e^{-2\phi_h}, \end{aligned} \tag{3.6}$$

where ϕ_h is the value of the dilaton at the horizon. The specific heat of the hole is positive indicating that loosing mass through the Hawking radiation the hole will set down to its ground state which we have identified with ADS^0 .

At this stage a careful analysis of the mass spectrum of the solutions (3.2) is necessary particularly in view of the discussion of the Hawking evaporation process which will be the subject of the next section. The mass spectrum is labelled by a continuous parameter M , which in principle can be an arbitrary real number. From a two-dimensional point of view there is no reason to exclude the states with $M < 0$. In fact, differently from e.g. Schwarzschild black hole in 4d, the states with $M < 0$ do not correspond to naked singularities of the spacetime. The metrics (3.2) describe spacetime with constant curvature

for every value of a^2 . The separation of the spectrum in states with positive and negative mass is a consequence of the choice of ADS^0 as the ground state. This choice is from a two-dimensional point of view again arbitrary, since the mass spectrum is in principle unbounded and the system has no ground state.

It is interesting to compare this situation with that of the three-dimensional BTZ black holes [8,9]. The three-dimensional black holes (with angular momentum $J = 0$) have a continuous mass spectrum for $M > 0$ and the vacuum is regarded as the empty space obtained by letting the horizon size go to zero:

$$ds_{vac}^2 = -(\Lambda r)^2 dt^2 + (\Lambda r)^{-2} dr^2 + r^2 d\theta^2. \quad (3.7)$$

For $M < 0$ one has solutions which describe naked singularities, unless $M = -1$. In this case the metric describes the three-dimensional anti-de Sitter space :

$$ds^2 = -[(\Lambda r)^2 + 1] dt^2 + [(\Lambda r)^2 + 1]^{-1} dr^2 + r^2 d\theta^2, \quad (3.8)$$

One sees that the three-dimensional anti-de Sitter space emerges as bound state, separated from the continuous black hole spectrum by a mass gap of one unit, i.e. by a sequence of naked singularities which of course cannot be included in the configuration space. The dimensional reduction from three to two dimensions not only makes all spacetimes of constant negative curvature locally equivalent up to a coordinate transformation, but also eliminates the mass gap in the spectrum making it continuous and unbounded.

Let us now explain how one can single out ADS^0 as the physical ground state for the two-dimensional theory. The point is again the relationship between the two-dimensional theory and the four-dimensional one described in the previous section. In fact the two-dimensional black hole mass M is related to the parameter Δ in eq. (2.11) which measures the deviation from extremality of the four-dimensional black hole. For $M < 0$ we have $\Delta < 0$ with negative r_- and r_+ , the singularity at $r = 0$ becomes visible and the solutions (2.4) describe naked singularities. Therefore if we want our two-dimensional theory to describe nearly extreme four-dimensional black holes we have to consider only the $M \geq 0$ range as the physical mass spectrum for our two-dimensional black hole.

Let us now study the response of our two-dimensional dilaton-gravity system to the introduction of matter. Consider N massless fields f_i conformally coupled to the 2d-gravity model defined by the action (3.1). The classical action is:

$$S = \frac{1}{2\pi} \int d^2x \sqrt{g} \left[e^{-2\phi} (R + 2\Lambda) - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right]. \quad (3.9)$$

In the conformal gauge

$$ds^2 = -e^{2\rho} dx^+ dx^-, \quad x^\pm = x^0 \pm x^1,$$

the equation of motion and the constraints are

$$\begin{aligned} \partial_+ \partial_- f_i &= 0, \\ \partial_+ \partial_- \rho &= -\frac{\Lambda^2}{4} e^{2\rho}, \\ \partial_+ \partial_- e^{-2\phi} &= -\frac{\Lambda^2}{2} e^{2(\rho-\phi)}, \\ \partial_+^2 e^{-2\phi} - 2\partial_+ \rho \partial_+ e^{-2\phi} &= -\frac{1}{2} \sum_{i=1}^N \partial_+ f_i \partial_+ f_i, \\ \partial_-^2 e^{-2\phi} - 2\partial_- \rho \partial_- e^{-2\phi} &= -\frac{1}{2} \sum_{i=1}^N \partial_- f_i \partial_- f_i. \end{aligned} \quad (3.10)$$

The peculiarity of this system of differential equations is that the equation which determines the metric (the conformal factor ρ) is independent of both the dilaton and the matter fields. As a consequence one can solve independently the equation for ρ , the dilaton being determined afterwards by the expression for the metric and, through the constraints, by the stress energy tensor of the matter. This analysis has been carried out in [6], where it was demonstrated that the vacuum solutions ($f_i = 0$) of (3.10) can be written, using the residual coordinate invariance within the conformal gauge, as

$$e^{2\rho} = \frac{4}{\Lambda^2} (x^- - x^+)^{-2}, \quad (3.11)$$

$$e^{2(\phi-\phi_0)} = \frac{\Lambda}{2} (x^- - x^+), \quad (3.12)$$

which represent ADS^0 in the conformal gauge. Here and in the following we will consider only the region $x^- \geq x^+$ of the spacetime which corresponds to the $r \geq 0$ region in Schwarzschild coordinates. The solutions generated by a f -shock wave with stress tensor

$$T_{++} = 2\alpha^2 \Lambda^{-1} e^{-2\phi_0} \delta(x^+ - x_0^+), \quad (3.13)$$

are instead

$$e^{2\rho} = \frac{4}{\Lambda^2} (x^- - x^+)^{-2}, \quad (3.14)$$

$$e^{2(\phi - \phi_0)} = \frac{\Lambda}{2} \frac{(x^- - x^+)}{1 - \alpha^2 x^+ x^-}. \quad (3.15)$$

The constant α is related to the ADM mass of the solution by $M = e^{-2\phi_0} \Lambda^{-1} \alpha^2 / 4$.

The effect of the shock wave on the vacuum solution (3.11),(3.12) is therefore encoded in the modification (3.15) of the dilaton, the metric part (3.14) of the solution being insensitive to the presence of matter. However this is true only if one chooses to take a maximal extension of the spacetime (3.14), i.e. if one continues analytically the solution beyond the line $1 - \alpha^2 x^+ x^- = 0$ (the point $r = 0$ in Schwarzschild coordinates), where $\exp(-2\phi)$ becomes negative. As we have seen previously the interpretation of ADS^+ as a black hole can be established only if one cuts off the spacetime at this line. Therefore even though the local properties of the metric (3.14) are insensitive to the presence of matter, the global ones (the topology) are not; the effect of the shock wave on the vacuum (3.11),(3.12) is to create a boundary of the spacetime along the line where $\exp(-2\phi)$ becomes negative.

The analysis of the dynamical evolution of our two-dimensional system in this general setting could be implemented by imposing some appropriate boundary conditions for the fields and then by considering the dynamics of the boundary. In this paper we are mainly interested in the semiclassical properties of the black hole solution such as the Hawking evaporation process: it turns out that for the study of such process an exact knowledge of the dynamics of the boundary is quite unnecessary. In fact, since the Hawking evaporation is a process which takes place outside the event horizon, to study it is enough to consider a coordinate system in which the boundary at $r = 0$ is not visible. This can be done in a standard way starting from eq (3.2), defining the "Regge-Wheeler tortoise" coordinate r_*

and light-cone coordinates σ^+, σ^- as follows:

$$r_* = -\frac{1}{a\Lambda} \operatorname{arctanh} \left(\frac{\Lambda r}{a} \right), \quad \sigma^+ = t + r_*, \quad \sigma^- = t - r_*. \quad (3.16)$$

In these coordinates, the solution (3.2) with $a^2 > 0$ becomes

$$\begin{aligned} ds^2 &= -a^2 \sinh^{-2} \left[(\sigma^- - \sigma^+) \frac{a\Lambda}{2} \right], \\ e^{2(\phi - \phi_0)} &= \frac{1}{a} \tanh \left[(\sigma^- - \sigma^+) \frac{a\Lambda}{2} \right]. \end{aligned} \quad (3.17)$$

which represents ADS^+ in the conformal gauge. The new coordinate system covers only the region $r > r_h$ of the spacetime defined by (3.2). The metric part of the solution (3.17) can be brought into the form (3.14) by the transformation

$$\begin{aligned} x^+ &= \frac{2}{a\Lambda} \tanh(a\Lambda\sigma^+/2) \\ x^- &= \frac{2}{a\Lambda} \tanh(a\Lambda\sigma^-/2). \end{aligned} \quad (3.18)$$

From the previous equations one easily sees that the coordinates σ^+, σ^- cover only the region $\{-2/a\Lambda < x^+ < 2/a\Lambda, -2/a\Lambda < x^- < 2/a\Lambda\}$ of the spacetime defined by (3.11) which represent ADS^0 in the conformal gauge.

The coordinate transformation (3.18) gives the relationship between the coordinate system "free-falling" on the horizon of the black hole and the "anti-de Sitter" asymptotic coordinate system. This relationship is similar to that between a Rindler and a Minkowski coordinate system in two-dimensional flat spacetime even though in our case the coordinate transformation does not correspond to the motion of any physical observer. Moreover in our case the presence of the boundary at $r = 0$, which makes ADS^+ and ADS^0 topologically not equivalent, has a dynamical interpretation in terms of the matter distribution in the spacetime. In this sense the JT theory has a very interesting and particular status in the context of the gravity theories in arbitrary spacetime dimensions. In fact the distribution of matter does not determine the local geometry of the space, as usually happens for gravity theories, but its global properties, i.e. its topology. At first sight this conclusion seems to hold only for the conformal matter-gravity coupling defined by the action (3.9). A more general coupling, for example a dilaton-dependent coupling of the matter, could

spoil this property. In fact as a result of such a coupling a direct relationship between the local geometry and the distribution of matter would be at hand. However such a coupling cannot have any impact on the feature which seems to be responsible for the particular behavior of the theory, namely the fact that the solutions of the field equation in absence of matter describe different parametrizations of the same space and can be therefore physically distinguished only through boundary conditions.

The features of our model are in some sense shared by all the two-dimensional dilaton gravity models. In fact it has been demonstrated that all the models of this type are physically equivalent to a model of free matter fields reflecting off a dynamical moving mirror [12]. In this context the JT theory represents the extreme case: the formulation of the model in terms of a dynamical boundary is the only possible because the dynamics in terms of the gravitational and dilaton field is in some sense trivial.

4. Semiclassical properties.

As discussed before, the difference between the ground state and the black hole spacetime is essentially of topological nature. As is well known, different topologies of spacetimes with the same geometrical background give rise to different vacuum states for quantum fields and by consequence the vacuum in one spacetime will be perceived as a thermal bath of radiation by an observer in the other system.

A typical example of this is the quantization of scalar fields in a Rindler spacetime. This spacetime describes flat space as seen by a uniformly accelerated observer. A transformation of coordinates exists which puts the Rindler line element into the Minkowski form: however the coordinates so defined do not cover the whole Minkowski spacetime (physically, an accelerated observer cannot see the whole spacetime, but a horizon hides part of it to his view). It follows that two different Hilbert spaces are necessary for the quantization of fields in Rindler and Minkowski spaces: the vacuum state in one spacetime will be perceived as a thermal state in the other.

Our case is analogous: the ADS^+ black hole is related by a coordinate transformation to the ADS^0 ground state, but its image does not cover the whole ADS^0 : for this reason it is not possible to define the same vacuum state for the two spacetimes and a thermal

radiation will be observed.

Two main differences are however present in our case: first of all, the coordinate transformation between ADS^+ and ADS^0 does not correspond to the motion of any physical observer, so that the two spacetime should be considered as physically distinct, and not as the same spacetime seen from different observers. Second, the anti-de Sitter spacetime has a large group of symmetries, so that the coordinate transformation (3.18) between (3.11) and (3.17) is not uniquely defined, as we shall see in the following. Of course, this fact has no consequence on the physical results, which are independent on the choice of the transformation.

Finally, we recall that, when defining quantum field theory on anti-de Sitter, some caution should be taken with the boundary conditions at infinity, since such spacetime is not globally hyperbolic [13]. In the following, as explained in detail in [6], we shall use "transparent" boundary conditions, since they are more suitable for the physical process at hand.

In ref. [6] we considered already this case, but we neglected the difference in the global properties of ADS^0 and ADS^+ : the answer we obtained was of course that no Hawking radiation at all can be detected. This confirms the purely topological nature of the Hawking radiation in this model.

The ground state for the Hawking radiation is given, if one excludes from the theory the negative mass states, by the ADS^0 spacetime with metric (3.11), while the positive mass black hole is described by a metric of the form (3.17). In the following discussion, however, we shall define the σ^\pm coordinates in a different way from (3.18). This is possible because the $GL(2, R)$ group of transformation :

$$x^+ \rightarrow \frac{ax^+ + b}{cx^+ + d}, \quad x^- \rightarrow \frac{ax^- + b}{cx^- + d}, \quad (4.1)$$

with $ad - bc \neq 0$ is the isometry group of the metric (3.11), so that one can define different coordinates which lead to the same metric form: $GL(2, R)$ should therefore be considered as the invariance group of the ADS^0 vacuum. Thus, we define σ^\pm such that:

$$x^- = \frac{1}{a\Lambda} e^{a\Lambda\sigma^-}, \quad x^+ = \frac{1}{a\Lambda} e^{a\Lambda\sigma^+}, \quad (4.2a)$$

in F and

$$x^- = -\frac{1}{a\Lambda}e^{a\Lambda\sigma^-}, \quad x^+ = -\frac{1}{a\Lambda}e^{a\Lambda\sigma^+}, \quad (4.2b)$$

in P , where the F and P regions respectively correspond to positive or negative x^+ and are displayed in fig. 5: ADS^+ covers the $x^+x^- \geq 0$ region of ADS^0 and the black hole horizon corresponds to $x^+x^- = 0$.

The physical meaning of the coordinate transformation (4.2) looks clearer if one uses the standard spacetime coordinates : $x = (x^- - x^+)/2, t = (x^- + x^+)/2, \sigma = (\sigma^- - \sigma^+)/2, \tau = (\sigma^- + \sigma^+)/2$. In these coordinates , ADS^0 is given by:

$$ds^2 = \frac{1}{\Lambda^2 x^2}(-dt^2 + dx^2), \quad (4.3)$$

with $-\infty < t < \infty, 0 < x < \infty$ and the ADS^+ black hole has line element:

$$ds^2 = \frac{a^2}{\sinh^2(a\Lambda\sigma)}(-d\tau^2 + d\sigma^2), \quad (4.4)$$

with $-\infty < \tau < \infty, 0 < \sigma < \infty$. The two metrics are related by the change of variable:

$$\begin{aligned} x &= \pm \frac{1}{a\Lambda} e^{a\Lambda\tau} \sinh(a\Lambda\sigma), \\ t &= \pm \frac{1}{a\Lambda} e^{a\Lambda\tau} \cosh(a\Lambda\sigma). \end{aligned} \quad (4.5)$$

The change of variable (4.5), as already noticed, does not correspond to the motion of a physical observer, since its trajectory would be space-like. By consequence, the causal structure is different from that encountered in the Rindler problem. However, the mathematical structure is identical and for this reason we prefer to adopt these coordinates instead of the more intuitive ones defined by (3.18).

Let us consider the quantization of a single massless scalar field f in the fixed background defined by ADS^0 and ADS^+ . In ADS^0 , f can be expanded in terms of the basis:

$$\bar{u}_k = \frac{1}{\sqrt{4\pi\omega}} e^{ikx - i\omega t}, \quad (4.6)$$

with $\omega = |k|$ and $-\infty < k < \infty$. The $k > 0$ modes are left-moving waves:

$$\frac{1}{\sqrt{4\pi\omega}} e^{-i\omega x^+},$$

while $k < 0$ corresponds to right-moving waves

$$\frac{1}{\sqrt{4\pi\omega}} e^{-i\omega x^-}.$$

These two sets are positive frequency with respect to the Killing vector ∂_t .

Analogously, one can define a basis in the black hole regions P and F :

$${}^F u_k = \frac{1}{\sqrt{4\pi\omega}} e^{ik\sigma - i\omega\tau} \quad \text{in } F; \quad 0 \quad \text{in } P; \quad (4.7a)$$

and

$${}^P u_k = \frac{1}{\sqrt{4\pi\omega}} e^{ik\sigma + i\omega\tau} \quad \text{in } P; \quad 0 \quad \text{in } F; \quad (4.7b)$$

with $\omega = |k|$, which are positive frequency with respect to ∂_τ .

This basis can be continued to the whole $x - t$ plane. However, due to the change of sign at $x^+ = x^- = 0$, it is not analytic at that point. Consequently, it defines an alternative Fock space, which corresponds to a vacuum state different from that defined by the basis (4.6). By a standard argument [14], the ADS^+ vacuum state will therefore appear to an observer in the ADS^0 vacuum as filled of thermal radiation. The actual content can be obtained by calculating the Bogoliubov coefficients between the two vacua. To this end, it is useful to define some linear combinations of the ADS^+ basis:

$${}^F u_k + e^{\frac{2\pi\omega}{a\Lambda}} {}^P u_{-k}^* \quad (4.8a),$$

and

$${}^F u_{-k}^* + e^{\frac{2\pi\omega}{a\Lambda}} {}^P u_k. \quad (4.8b)$$

Contrary to (4.7), this basis is analytic for all real values of x^+ and x^- and hence shares the same vacuum state with the ADS^0 basis (4.6). In fact (4.8a) is proportional to

$$(\bar{u}_k)^{-i\frac{\omega}{a\Lambda}} \quad \text{for } k > 0, \quad (\bar{v}_k)^{-i\frac{\omega}{a\Lambda}} \quad \text{for } k < 0,$$

while (4.8b) is proportional to

$$(\bar{v}_k)^{i\frac{\omega}{a\Lambda}} \quad \text{for } k > 0, \quad (\bar{u}_k)^{i\frac{\omega}{a\Lambda}} \quad \text{for } k < 0.$$

Comparing the expansion of f in the two basis:

$$\begin{aligned}
f &= b_k^{(1) \ P} u_k + b_k^{(1)\dagger \ P} u_k^* + b_k^{(2) \ F} u_k + b_k^{(2)\dagger \ F} u_k^* = \\
&= \frac{1}{\sqrt{2 \sinh \frac{2\pi\omega}{a\Lambda}}} [d_k^{(1)} \left(e^{\frac{\pi\omega}{2a\Lambda}} F u_k + e^{-\frac{\pi\omega}{2a\Lambda}} P u_{-k}^* \right) + \\
&\quad + d_k^{(2)} \left(e^{\frac{\pi\omega}{2a\Lambda}} F u_{-k}^* + e^{-\frac{\pi\omega}{2a\Lambda}} P u_k \right) + \text{h.c.}],
\end{aligned} \tag{4.9}$$

where the operators $d_k^{(i)}$ annihilate the ADS^0 vacuum state $|0_0\rangle$, while the $b_k^{(i)}$ annihilate the black hole vacuum state $|0_+\rangle$, one gets

$$\begin{aligned}
b_k^{(1)} &= \frac{1}{\sqrt{2 \sinh \frac{2\pi\omega}{a\Lambda}}} \left[e^{\frac{\pi\omega}{2a\Lambda}} d_k^{(2)} + e^{-\frac{\pi\omega}{2a\Lambda}} d_k^{(1)\dagger} \right] \\
b_k^{(2)} &= \frac{1}{\sqrt{2 \sinh \frac{2\pi\omega}{a\Lambda}}} \left[e^{\frac{\pi\omega}{2a\Lambda}} d_k^{(1)} + e^{-\frac{\pi\omega}{2a\Lambda}} d_k^{(2)\dagger} \right]
\end{aligned}$$

from which one can read off the Bogoliubov coefficients.

In particular, an observer in the ADS^0 vacuum detects a thermal flux of particles with spectrum:

$$\langle_{+} 0 | d_k^\dagger d_k | 0_+ \rangle = \frac{1}{e^{\frac{2\pi\omega}{a\Lambda}} - 1}. \tag{4.10}$$

When integrated, it gives for the total flux of f -particle energy:

$$G = \frac{a^2 \Lambda^2}{48}. \tag{4.11}$$

The flux corresponds to a Planck spectrum at temperature $T_0 = \frac{a\Lambda}{2\pi}$, the Hawking temperature (3.6) of the black hole. The local temperature at a given point is instead:

$$T = (g_{00})^{-\frac{1}{2}} T_0 = \frac{\Lambda}{2\pi} \sinh(a\Lambda\sigma)$$

which goes to zero at spatial infinity. As a check of the previous result we can compute the outgoing stress tensor for the Hawking radiation in terms of the relationship (3.18) between the coordinates σ and x . In general it is proportional to the Schwarzian derivative of the function $x^- = F(\sigma^-)$:

$$\langle 0 | T_{--} | 0 \rangle = -\frac{1}{24} \left[\frac{F'''}{F'} - \frac{3}{2} \left(\frac{F''}{F'} \right)^2 \right],$$

where the primes indicate derivation with respect to σ^- . With F given by (3.18) and making use of (3.3) we get

$$\langle 0|T_{--}|0\rangle = \frac{1}{48}a^2\Lambda^2 = \frac{1}{24}e^{-2\phi_0}M\Lambda. \quad (4.12)$$

in accordance with eq. (4.11). The stress energy tensor for the Hawking radiation has therefore the constant (thermal) value which one would naively expect in view of the specific heat expression (3.6) and coincides with the $k = 0$ limit of the result obtained in [6]. Of course, we are not considering the back reaction of the metric, so that the flux is independent of time. In the real physical process the mass and hence the radiation rate, will decrease as the black hole radiates until the total evaporation. This means that in this approximation, loosing mass through the radiation the black hole will settle down to its ground state $M = 0$ which is represented by ADS^0 vacuum. It is interesting to note that the stress tensor of the Hawking radiation has the same invariance group of the schwarzian derivative, i.e the $GL(2, R)$ group realised as the fractional transformation (4.1), which is also the invariance group of ADS^0 , as discussed before.

Let us now discuss the inclusion in our calculations of the back reaction of the radiation on the gravitational background. We can guess in view of the general features of our model that this effect is not related to a change of the geometry of the spacetime but to a change of the boundary conditions. In fact the classical solutions of our theory are distinguishable only by means of global properties and the Hawking radiation is a purely topological effect. The back reaction of the radiation on the metric can be studied in a standard way by considering, in the quantization of the scalar fields f , the contribution of the trace anomaly to the effective action. This contribution is the well-known Polyakov-Liouville action which in the conformal gauge is a local term. The semiclassical action in the conformal gauge is

$$S = \frac{1}{\pi} \int d^2\sigma \left[\left(2\partial_+\partial_-\rho + \frac{\Lambda^2}{2}e^{2\rho} \right) e^{-2\phi} + \frac{1}{2} \sum_{i=1}^N \partial_+f_i\partial_-f_i - \frac{N}{12}\partial_+\rho\partial_-\rho \right]. \quad (4.13)$$

Where the first three terms come from the classical action (3.9) whereas the fourth describes the trace anomaly. The determinant of the kinetic energies in the (ρ, ϕ) space is

proportional to $\exp(-4\phi)$ so that there is no degeneration of the kinetic energies in the physical field space ($\exp(-2\phi) > 0$). This behaviour is to be compared with the CGHS dilaton gravity theory where the kinetic energies become degenerate at the point of the field space where $\exp(-2\phi) = N/12$ [15]. The ensuing equation of motion and constraint are

$$\begin{aligned}
\partial_+ \partial_- f_i &= 0, \\
\partial_+ \partial_- \rho &= -\frac{\Lambda^2}{4} e^{2\rho}, \\
\partial_+ \partial_- e^{-2\phi} &= \frac{\Lambda^2}{2} e^{2\rho} \left(\frac{N}{24} - e^{-2\phi} \right), \\
\partial_+^2 e^{-2\phi} - 2\partial_+ \rho \partial_+ e^{-2\phi} &= -\frac{1}{2} \sum_{i=1}^N \partial_+ f_i \partial_+ f_i + \frac{N}{12} \left[(\partial_+ \rho)^2 - \partial_+^2 \rho + t_+ \right], \\
\partial_-^2 e^{-2\phi} - 2\partial_- \rho \partial_- e^{-2\phi} &= -\frac{1}{2} \sum_{i=1}^N \partial_- f_i \partial_- f_i + \frac{N}{12} \left[(\partial_- \rho)^2 - \partial_-^2 \rho + t_- \right],
\end{aligned} \tag{4.14}$$

where the functions t_+, t_- must be determined using boundary conditions. The semiclassical equations of motion differ from the classical ones just in the shift

$$\exp(-2\phi) \rightarrow \exp(-2\phi) + N/24 \tag{4.15}$$

of the dilaton and for the presence in the constraint equation of the term stemming from the conformal anomaly. This result can be achieved directly from the action (4.13) by noting that the term describing the conformal anomaly can be reabsorbed in the classical action by means of the former redefinition of the dilaton. This means that the solutions of the semiclassical equations of motion are locally the same as the classical ones and differ from those only through the boundary conditions.

If one ignores the presence of the boundary on the spacetime along the line where $\exp(-2\phi)$ becomes negative, i.e. if one takes maximally extended solutions there is, as seen previously, no Hawking radiation and therefore no back reaction. On the other hand when the boundary is present and supported by appropriate boundary conditions the problem of the back reaction can in principle be analysed by studying, together with eq.(4.14), the dynamical evolution of the boundary in the context of the semiclassical theory. We will not address the problem in this general setting, but will study it in a static approximation

which is however consistent with our previous treatment of the Hawking radiation process. Let us consider the process in which the black hole radiates as a succession of static states with decreasing mass. Being the solutions static they will depend only on the variable $\sigma = \sigma^- - \sigma^+$.

The equations of motion and the constraint (4.14), with $f_i = 0$, become now:

$$\begin{aligned}\rho'' &= \frac{\Lambda^2}{4} e^{2\rho}, \\ \psi'' &= \frac{\Lambda^2}{2} e^{2\rho} \left(\psi - \frac{N}{24} \right), \\ \psi'' - 2\rho'\psi' &= \frac{N}{12} [(\rho')^2 - \rho''],\end{aligned}\tag{4.16}$$

where $\psi = \exp(-2\phi)$ and the primes represent derivatives with respect to σ . Integration of the equations of motion (4.16) gives two class of solution (we do not consider the solutions which correspond to ADS^-)

$$\begin{aligned}e^{2\rho} &= \frac{4c^2}{\Lambda^2} \sinh^{-2}(c\sigma), \\ e^{-2\phi} &= \left(e^{-2\phi_0} + \frac{N}{24} c\sigma \right) \tanh^{-1}(c\sigma),\end{aligned}\tag{4.17}$$

and

$$\begin{aligned}e^{2\rho} &= \frac{4}{\Lambda^2 \sigma^2}, \\ e^{-2\phi} &= \frac{e^{-2\phi_0}}{\sigma} + \frac{N}{24}.\end{aligned}\tag{4.18}$$

In eq. (4.17) c is an integration constant, for $c = a\Lambda/2$ the metric part of the solution coincide with the classical solution (3.17), it describes therefore ADS^+ . The solution (4.18) can be considered the ground state of the semiclassical theory and coincides with the ground state of the classical theory (ADS^0) after the shift (4.15) of the dilaton. Let us now discuss the semiclassical solutions (4.17), (4.18). First we note that for $e^{2\phi} \ll 24/N$ the semiclassical solutions (4.17),(4.18) behave as the classical ones (3.11),(3.17). Hence in the weak coupling regime we can safely ignore the back reaction. Moreover, one can easily realize that the back reaction affects only the dilaton but not the metric. This is again a feature which is connected with the purely topological nature of the Hawking radiation. The states with decreasing values of c or equivalently of the black hole mass, in eq (4.17)

describe the evolution of the black hole when the evaporation process takes place if we think of it as a succession of static states. In particular, the limit $c \rightarrow 0$ ($M \rightarrow 0$) will tell us what is the end of the evaporation process. Performing the limit $c \rightarrow 0$ in the solution (4.17) we get the vacuum solution (4.18). Thus the end point of the evaporation process is exactly the ADS^0 . In a pictorial description of the process we see that the region of the ADS^0 spacetime covered by the coordinate σ increases (equivalently the horizon of the black hole recedes) as the black hole loses mass through the radiation and, at the end point, this region coincides with the whole ADS^0 space (see figure 6).

5. Summary and outlook.

We have studied the four-dimensional analog of the scalar-gravity JT theory and shown that it admits regular, asymptotically flat magnetically charged solutions which can be interpreted as non-singular black holes .

The final state of the evaporation process of these black holes is given, as usual, by the extremal limit of the metric. The evaporation can be described by means of a two-dimensional effective theory which is governed by the JT action. This theory admits anti-de Sitter solutions which can be interpreted as the conformal vacuum or as black holes depending on the boundary conditions one chooses. These are determined, through the form of the dilaton, by the four-dimensional physics we want to describe with the two-dimensional model.

We have calculated the flux of Hawking radiation in a way analogous to that used in the case of Rindler spacetime and also by computing the energy-momentum tensor. Moreover, by means of a static approximation, we have discussed the inclusion of the back reaction of the fields to the radiation, arguing that it results in a smooth evolution of the black hole towards the ground state , in contrast with the CGHS case , where some singularities appear in the evolution.

A more careful analysis should involve the dynamical evolution of the boundary conditions . It is not clear at the moment whether this can be deduced from the lagrangian or, owing to the global nature of the problem, some further input should be added to the formalism.

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